**Universidade Federal do Vale do São Francisco**
**Engenharia Civil**
**Métodos Matemáticos**

**Prof$^{o}$. Edson**

**2 º Semestre**

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**Exercício 1** *Considere* $z = x + \mathbf{i}y$ *um número complexo de módulo* $\sqrt{3}$ *e que está sobre a reta* $y = 2x + 1$*, ou seja*

$$\begin{cases} x^2 + y^2 = 3 \\ y = 2x + 1 \end{cases}$$

*Substituindo a segunda equação deste sistema, na primeira tem-se*

$$x^2 + (2x+1)^2 = 3 \Rightarrow$$

$$5x^2 + 4x - 2 = 0$$

*Ou seja,*

$$x = -\frac{2}{5} \pm \frac{\sqrt{14}}{5}$$

*e*

$$y = \frac{1}{5} \pm \frac{2\sqrt{14}}{5}$$

*Portanto*

$$z = \left(-\frac{2}{5} + \frac{\sqrt{14}}{5}\right) + \left(\frac{1}{5} + \frac{2\sqrt{14}}{5}\right)\mathbf{i}$$

*ou*

$$z = \left(-\frac{2}{5} - \frac{\sqrt{14}}{5}\right) + \left(\frac{1}{5} - \frac{2\sqrt{14}}{5}\right)\mathbf{i}$$

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**Exercício 2** *Observe que*

$$\begin{aligned}
\left[(1-\mathbf{i})^3 + \mathbf{i}^{157}\right]\left(1+\mathbf{i}^5\right)^{-1} &= \frac{(1-\mathbf{i})^3 + \mathbf{i}^{157}}{1+\mathbf{i}^5} \\
&= \frac{-2\mathbf{i}(1-\mathbf{i}) + \mathbf{i}^{2\times 78+1}}{1+\mathbf{i}} \\
&= \frac{-2-2\mathbf{i} + (-1)^{78}\,\mathbf{i}}{1+\mathbf{i}} \\
&= \frac{-2-\mathbf{i}}{1+\mathbf{i}}\,\frac{1-\mathbf{i}}{1-\mathbf{i}} \\
&= \frac{-3+\mathbf{i}}{2} \\
&= -\frac{3}{2} + \frac{1}{2}\mathbf{i}
\end{aligned}$$

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**Exercício 3** *Deseja-se encontrar todas as raízes da equação*

$$w^6 + w^3 + 1 = 0$$

*Para isso, considere*

$$z = w^3$$

*e observe a equação dada, pode ser reescrita como*

$$z^2 + z + 1 = 0$$

*Donde segue-se que*

$$z = \frac{-1 + \mathbf{i}\sqrt{3}}{2}$$

*ou*

$$z = \frac{-1 - \mathbf{i}\sqrt{3}}{2}$$

*Na variável* $w$*,*

$$w^3 = \frac{-1+\mathbf{i}\sqrt{3}}{2} \Rightarrow w = \left(\frac{-1+\mathbf{i}\sqrt{3}}{2}\right)^{\frac{1}{3}}$$

*ou*

$$w^3 = \frac{-1-\mathbf{i}\sqrt{3}}{2} \Rightarrow w = \left(\frac{-1-\mathbf{i}\sqrt{3}}{2}\right)^{\frac{1}{3}}$$

*Na forma polar,*

$$\begin{aligned}
w &= \left(-\frac{1}{2} + \frac{\mathbf{i}\sqrt{3}}{2}\right)^{\frac{1}{3}} \\
&= \left(\cos\frac{2\pi}{3} + \mathbf{i}\,\mathrm{sen}\frac{2\pi}{3}\right)^{\frac{1}{3}}
\end{aligned}$$

*Logo*

$$w_0 = \cos\frac{2\pi}{9} + \mathbf{i}\,\mathrm{sen}\frac{2\pi}{9}$$

$$w_1 = \cos\frac{8\pi}{9} + \mathbf{i}\,\mathrm{sen}\frac{8\pi}{9}$$

$$w_2 = \cos\frac{14\pi}{9} + \mathbf{i}\,\mathrm{sen}\frac{14\pi}{9}$$

*Do mesmo modo,*

$$\begin{aligned} w &= \left(-\frac{1}{2} - \frac{\mathbf{i}\sqrt{3}}{2}\right)^{\frac{1}{3}} \\ &= \left(\cos\frac{4\pi}{3} + \mathbf{i}\,\mathrm{sen}\frac{4\pi}{3}\right)^{\frac{1}{3}} \end{aligned}$$

*e,*

$$w_3 = \cos\frac{4\pi}{9} + \mathbf{i}\,\mathrm{sen}\frac{4\pi}{9}$$

$$w_4 = \cos\frac{10\pi}{9} + \mathbf{i}\,\mathrm{sen}\frac{10\pi}{9}$$

$$w_5 = \cos\frac{16\pi}{9} + \mathbf{i}\,\mathrm{sen}\frac{16\pi}{9}$$

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**Exercício 4** *Para que a função dada seja contínua em $z = 3\mathbf{i}$, é necessário que*

$$\begin{aligned} f(3\mathbf{i}) &= \lim_{z\to 3\mathbf{i}} f(z) \\ &= \lim_{z\to 3\mathbf{i}} \frac{z^4 + 10z^2 + 9}{z^2 - 4\mathbf{i}z - 3} \\ &= \lim_{z\to 3\mathbf{i}} \frac{\left(z^3 + 3\mathbf{i}z^2 + z + 3\mathbf{i}\right)(z - 3\mathbf{i})}{(z-\mathbf{i})(z-3\mathbf{i})} \\ &= \lim_{z\to 3\mathbf{i}} \frac{z^3 + 3\mathbf{i}z^2 + z + 3\mathbf{i}}{z - \mathbf{i}} \\ &= \frac{-48\mathbf{i}}{2\mathbf{i}} \\ &= -24 \end{aligned}$$

*De modo semelhante, para que $f$ seja contínua em $z = \mathbf{i}$, é necessário que*

$$\begin{aligned} f(\mathbf{i}) &= \lim_{z\to \mathbf{i}} f(z) \\ &= \lim_{z\to \mathbf{i}} \frac{z^4 + 10z^2 + 9}{z^2 - 4\mathbf{i}z - 3} \\ &= \lim_{z\to \mathbf{i}} \frac{\left(z^3 + \mathbf{i}z^2 + 9z + 9\mathbf{i}\right)(z - \mathbf{i})}{(z-\mathbf{i})(z-3\mathbf{i})} \\ &= \lim_{z\to \mathbf{i}} \frac{z^3 + \mathbf{i}z^2 + 9z + 9\mathbf{i}}{z - 3\mathbf{i}} \\ &= \frac{16\mathbf{i}}{-2\mathbf{i}} \\ &= -8 \end{aligned}$$

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**Exercício 5** *Deseja-se calcular o limite*

$$\lim_{z\to -2\mathbf{i}} \frac{z^3 - 8\mathbf{i}}{z + 2\mathbf{i}}$$

*Para isto, perceba antes que*

$$z^3 - 8\mathbf{i} = (z + 2\mathbf{i})\left(z^2 - 2\mathbf{i}z - 4\right)$$

*Logo,*

$$\begin{aligned} \lim_{z\to -2\mathbf{i}} \frac{z^3 - 8\mathbf{i}}{z + 2\mathbf{i}} &= \lim_{z\to -2\mathbf{i}} \frac{(z + 2\mathbf{i})\left(z^2 - 2\mathbf{i}z - 4\right)}{z + 2\mathbf{i}} \\ &= \lim_{z\to -2\mathbf{i}} \left(z^2 - 2\mathbf{i}z - 4\right) \\ &= -12 \end{aligned}$$

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