Universidade Federal do Vale do São Francisco
Engenharia Civil
Métodos Matemáticos

Prof$^{\text{o}}$. Edson

Semestre Suplementar

Gabarito 1ª Prova **2020**
Data: Domingo, 29 de Novembro **Turma EX**

Exercício 1 *Considere*

$$z = \left[\left(-8 + 8\sqrt{3}\mathbf{i}\right)(-1 - \mathbf{i})\right]^2$$

Observe que

$$\begin{aligned} z &= \left(-8 + 8\sqrt{3}\mathbf{i}\right)^2 (-1 - \mathbf{i})^2 \\ &= \left(64 - 128\sqrt{3}\mathbf{i} - 192\right)(1 + 2\mathbf{i} - 1) \\ &= 2\mathbf{i}\left(-128 - 128\sqrt{3}\mathbf{i}\right) \\ &= 256\sqrt{3} - 256\mathbf{i} \end{aligned}$$

donde segue-se que

$$\begin{aligned} |z| &= \left|256\left(\sqrt{3} - \mathbf{i}\right)\right| \\ &= |256|\left|\sqrt{3} - \mathbf{i}\right| \\ &= 256\sqrt{4} \\ &= 512 \end{aligned}$$

e

$$\begin{aligned} \arg z &= \operatorname{arctg}\left(\frac{-256}{256\sqrt{3}}\right) \\ &= \operatorname{arctg}\left(\frac{-1}{\sqrt{3}}\right) \\ &= -\frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z} \end{aligned}$$

Portanto,

$$\begin{aligned} z &= 512\left[\cos\left(-\frac{\pi}{6}\right) + \mathbf{i}\operatorname{sen}\left(-\frac{\pi}{6}\right)\right] \\ &= 512\left(\cos\frac{\pi}{6} - \mathbf{i}\operatorname{sen}\frac{\pi}{6}\right) \end{aligned}$$

$\blacksquare$

Exercício 2 *Inicialmente, perceba que*

$$-1 = \cos\pi + \mathbf{i}\operatorname{sen}\pi$$

Assim,

$$\begin{aligned} (-1)^{\frac{1}{3}} &= (\cos\pi + \mathbf{i}\operatorname{sen}\pi)^{\frac{1}{3}} \\ &= \cos\frac{\pi + 2k\pi}{3} + \mathbf{i}\operatorname{sen}\frac{\pi + 2k\pi}{3} \\ &= w_k \end{aligned}$$

onde $k = 0, 1, 2$. *Ou seja*

$$\begin{aligned} w_0 &= \cos\frac{\pi}{3} + \mathbf{i}\operatorname{sen}\frac{\pi}{3} \\ &= \frac{1}{2} + \mathbf{i}\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} w_1 &= \cos\frac{\pi + 2\pi}{3} + \mathbf{i}\operatorname{sen}\frac{\pi + 2\pi}{3} \\ &= \cos\pi + \mathbf{i}\operatorname{sen}\pi \\ &= -1 \end{aligned}$$

$$\begin{aligned} w_2 &= \cos\frac{\pi + 4\pi}{3} + \mathbf{i}\operatorname{sen}\frac{\pi + 4\pi}{3} \\ &= \frac{1}{2} - \mathbf{i}\frac{\sqrt{3}}{2} \end{aligned}$$

$\blacksquare$

Exercício 3 *Sabendo que* $z = 1$ *é raíz do polinômio*

$$p(z) = z^3 - 5z^2 + 17z - 13$$

segue-se que p *é divisível por* $z - 1$. *Realizando esta divisão tem-se*

$$p(z) = (z - 1)\left(z^2 - 4z + 13\right)$$

Ou seja, as outras raízes de p *são também raízes de*

$$q(z) = z^2 - 4z + 13$$

Para encontrá-las, observe que

$$\begin{aligned} \Delta &= 16 - 52 \\ &= -36 \end{aligned}$$

$$
\begin{aligned}
z &= \frac{4 \pm \sqrt{-36}}{2} \\
&= \frac{4 \pm 6\mathbf{i}}{2} \\
&= 2 \pm 3\mathbf{i}
\end{aligned}
$$

■

Exercício 4 *Sabe-se que*

$$f(z) = \frac{\mathrm{Im}(z)}{1 - |z|}$$

Observe então, que

i) $f(0) = \frac{\mathrm{Im}(0)}{1-|0|} = \frac{0}{1} = 0$. *Ou seja,* $f(0)$ *existe.*

ii) *Considerando* $z = x + \mathbf{i}y$ *é possível reescrever* f *da seguinte maneira*

$$f(z) = \frac{y}{1 - \sqrt{x^2 + y^2}}$$

ou seja

$$
\begin{aligned}
u(x,y) &= \frac{y}{1 - \sqrt{x^2 + y^2}} \\
v(x,y) &= 0
\end{aligned}
$$

e

$$
\begin{aligned}
\lim_{z \to 0} f(z) &= \lim_{(x,y)\to(0,0)} u(x,y) + \mathbf{i} \lim_{(x,y)\to(0,0)} v(x,y) \\
&= 0 + \mathbf{i}0 \\
&= 0
\end{aligned}
$$

iii) *Por fim, segue-se dos itens anteriores que*

$$\lim_{z \to 0} f(z) = f(0)$$

Portanto, a função dada é contínua em $z = 0$. ■

Exercício 5 *Considere*

$$w = (1 + z)^{\frac{1}{4}}$$

Observe que

$$
\begin{aligned}
w^4 &= 1 + z \Rightarrow \\
z &= w^4 - 1
\end{aligned}
$$

e,

$$z \to 0 \Rightarrow w \to 1^{\frac{1}{4}} = 1$$

Estamos admitindo aqui o ***valor principal*** *da raíz quarta. Segue-se, portanto, que*

$$\lim_{z \to 0} \frac{(1+z)^{\frac{1}{4}} - 1}{z} = \lim_{w \to 1} \frac{w - 1}{w^4 - 1}$$

Observe que

$$w^4 - 1 = (w - 1)\left(w^3 + w^2 + w + 1\right)$$

Assim,

$$
\begin{aligned}
\lim_{z \to 0} \frac{(1+z)^{\frac{1}{4}} - 1}{z} &= \lim_{w \to 1} \frac{w - 1}{(w-1)(w^3 + w^2 + w + 1)} \\
&= \lim_{w \to 1} \frac{1}{w^3 + w^2 + w + 1} \\
&= \frac{1}{4}
\end{aligned}
$$

■