**Universidade Federal do Vale do São Francisco**
**Mestrado Profissional em Matemática**
**Fundamentos de Cálculo**

**Prof$^{o}$. Edson**

**2$^{o}$ Semestre**

**Gabarito 1ª Prova** **2018**
**Data: Terça-feira, 10 de Junho** **Turma FC**

---

**Exercício 1** *Considere a função*

$$g(x) = f(x) - 5x$$

*Se*

$$f(0) = 0 \,\text{ou}\, f(1) = 5$$

*considere $\alpha = 0$ ou $\alpha = 1$, e está demonstrado o que se pede. Caso contrário, suponha*

$$f(0) > 0$$

$$f(1) < 5$$

*Desta forma, segue-se que*

$$g(0) = f(0) > 0$$

$$g(1) = f(1) - 5 < 0$$

*Como g é conínua em $[0,1]$, segue-se do teorema do valor intermediário que, existe $\alpha \in (0,1)$ tal que*

$$g(\alpha) = 0 \Rightarrow f(\alpha) - 5\alpha = 0 \Rightarrow f(\alpha) = 5\alpha$$

■

**Exercício 2** *Observe que*

$$\begin{aligned}
\operatorname{sen}\left(x^2 + \frac{1}{x}\right) - \operatorname{sen}\left(\frac{1}{x}\right) &= \operatorname{sen}\left(x^2\right)\cos\left(\frac{1}{x}\right) + \\
&\quad + \operatorname{sen}\left(\frac{1}{x}\right)\cos\left(x^2\right) - \\
&\quad - \operatorname{sen}\left(\frac{1}{x}\right) \\
&= \operatorname{sen}\left(x^2\right)\cos\left(\frac{1}{x}\right) + \\
&\quad + \operatorname{sen}\left(\frac{1}{x}\right)\left[\cos\left(x^2\right) - 1\right]
\end{aligned}$$

*Assim,*

$$\begin{aligned}
A &= \lim_{x\to 0}\left[\operatorname{sen}\left(x^2 + \frac{1}{x}\right) - \operatorname{sen}\left(\frac{1}{x}\right)\right] \\
&= \lim_{x\to 0}\left\{\operatorname{sen}\left(x^2\right)\cos\left(\frac{1}{x}\right) + \right. \\
&\quad \left. + \operatorname{sen}\left(\frac{1}{x}\right)\left[\cos\left(x^2\right) - 1\right]\right\}
\end{aligned}$$

*Sabendo que*

$$\left|\cos\left(\frac{1}{x}\right)\right| \leq 1$$

$$\left|\operatorname{sen}\left(\frac{1}{x}\right)\right| \leq 1$$

*e que*

$$\lim_{x\to 0}\operatorname{sen}\left(x^2\right) = 0$$

$$\lim_{x\to 0}\left[\cos\left(x^2\right) - 1\right] = 0$$

*Então,*

$$A = 0$$

■

**Exercício 3**

**a).**

$$\begin{aligned}
\lim_{x\to+\infty}\frac{x^2}{10 + x\sqrt{x}} &= \lim_{x\to+\infty}\frac{x^2}{x^2\left(\frac{10}{x^2} + \frac{1}{x}\sqrt{x}\right)} \\
&= \lim_{x\to+\infty}\frac{1}{\frac{10}{x^2} + \sqrt{\frac{1}{x^2}x}} \\
&= \lim_{x\to+\infty}\frac{1}{\frac{10}{x^2} + \sqrt{\frac{1}{x}}}
\end{aligned}$$

*Observe que*

$$\lim_{x\to+\infty}\left(\frac{10}{x^2} + \sqrt{\frac{1}{x}}\right) = 0^+$$

*Logo*

$$\lim_{x\to+\infty}\frac{x^2}{10 + x\sqrt{x}} = +\infty$$

□

**b).**

$$\lim_{x\to+\infty}\frac{\sqrt[3]{x^2+1}}{x+1}=\lim_{x\to+\infty}\frac{\sqrt[3]{x^3\left(\frac{1}{x}+\frac{1}{x^3}\right)}}{x\left(1+\frac{1}{x}\right)}$$

$$=\lim_{x\to+\infty}\frac{x\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}}{x\left(1+\frac{1}{x}\right)}$$

$$=\lim_{x\to+\infty}\frac{\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}}{1+\frac{1}{x}}$$

$$=0$$

□

**c).**

$$A=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x\left(1+\frac{1}{x}\sqrt{x+\sqrt{x}}\right)}}$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x}\sqrt{1+\frac{1}{x}\sqrt{\frac{1}{x^2}\left(x+\sqrt{x}\right)}}}$$

$$=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x}\sqrt{\frac{1}{x}+\frac{1}{x^2}\sqrt{x}}}}$$

$$=\lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x}\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x^3}}}}}$$

$$=1$$

■

**Exercício 4** *Seja* $f:\mathbb{R}\to\mathbb{R}$ *uma função tal que*

$$f(x+y)=f(x)+f(y),\ \forall x,y\in\mathbb{R}$$

*e suponha que*

$$\lim_{x\to 0}f(x)=L$$

**a).** *Inicialmente observe que*

$$\begin{aligned}f(2x)&=f(x+x)\\&=f(x)+f(x)\\&=2f(x)\end{aligned}$$

*Assim,*

$$\lim_{x\to0}f(2x)=\lim_{x\to0}2f(x)\Rightarrow$$

$$\lim_{x\to0}f(2x)=2\lim_{x\to0}f(x)$$

*Considere*

$$u=2x$$

*e perceba que*

$$x\to0\Rightarrow u\to0$$

*Ou seja,*

$$\lim_{x\to0}f\left(2x\right)=\lim_{u\to0}f\left(u\right)=L$$

*Portanto, segue-se que*

$$\begin{aligned}\lim_{x\to0}f(2x)&=2\lim_{x\to0}f(x)&\Rightarrow\\\lim_{u\to0}f\left(u\right)&=2\lim_{x\to0}f(x)&\Rightarrow\\L&=2L&\Rightarrow\\L&=0\end{aligned}$$

■

**b).** *Observe que*

$$x=(x-x_0)+x_0$$

*Logo*

$$f(x)=f\left(x-x_0\right)+f(x_0)$$

*e*

$$\lim_{x\to x_0}f(x)=\lim_{x\to x_0}\left[f\left(x-x_0\right)+f(x_0)\right]$$

$$=\lim_{x\to x_0}f\left(x-x_0\right)+\lim_{x\to x_0}f(x_0)$$

*Considere*

$$v=x-x_0$$

*e perceba que*

$$x\to x_0\Rightarrow v\to0$$

*Assim,*

$$\lim_{x\to x_0} f\,(x - x_0) = \lim_{v\to 0} f\,(v) = L$$

*Além disso,*

$$\lim_{x\to x_0} f(x_0) = f(x_0)$$

*Portanto*

$$\begin{aligned}\lim_{x\to x_0} f(x) &= \lim_{x\to x_0} f\,(x - x_0) + \lim_{x\to x_0} f(x_0)\\ &= L + f(x_0)\\ &= f(x_0)\end{aligned}$$

■

**Exercício 5** *Considere*

$$f(x) = x^n$$

*e observe que*

$$\begin{aligned}
f'(x) &= n\,x^{n-1} &&\Rightarrow\quad f'(1) = n\\
&&&= \frac{n!}{(n-1)!}\\
f''(x) &= n(n-1)x^{n-2} &&\Rightarrow\quad f''(1) = n(n-1)\\
&&&= \frac{n!}{(n-2)!}\\
f'''(x) &= n(n-1)(n-2)x^{n-3} &&\Rightarrow\quad f'''(1) = n(n-1)(n-2)\\
&&&= \frac{n!}{(n-3)!}\\
\cdots & &&\cdots\quad\cdots\\
f^{(n)}(x) &= n(n-1)\cdots 2\cdot 1\,x^0 &&\Rightarrow f^{(n)}(1) = n!\\
&&&= \frac{n!}{(n-n)!}
\end{aligned}$$

*Portanto*

$$\begin{aligned}
S &= f(1) + \frac{f'(1)}{1!} + \frac{f'(1)}{2!} + \cdots + \frac{f^{(n)}(1)}{n!}\\
&= \frac{n!}{1!(n-1)!} + \frac{n!}{2!(n-2)!} + \cdots + \frac{n!}{n!0!}\\
&= \sum_{i=1}^{n} \frac{n!}{i!(n-i)!}\\
&= \sum_{i=1}^{n} \binom{n}{i}\\
&= 2^n
\end{aligned}$$

■