Universidade Federal do Vale do São Francisco
Colegiado de Engenharia Civil
Cálculo Diferencial e Integral I

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2º Semestre

Gabarito 1ª Prova — **2018**
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Exercício 1

a). *Observe que*

$$\begin{aligned}
A &= \lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x} \\
&= \lim_{x\to 2} \frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x}\frac{\sqrt{x+2}+\sqrt{2x}}{\sqrt{x+2}+\sqrt{2x}} \\
&= \lim_{x\to 2} \frac{x+2-2x}{x\,(x-2)}\frac{1}{\sqrt{x+2}+\sqrt{2x}} \\
&= \lim_{x\to 2} \frac{-x+2}{x\,(x-2)}\frac{1}{\sqrt{x+2}+\sqrt{2x}} \\
&= \lim_{x\to 2} \frac{-\,(x-2)}{x\,(x-2)}\frac{1}{\sqrt{x+2}+\sqrt{2x}} \\
&= \lim_{x\to 2} \frac{-1}{x\left(\sqrt{x+2}+\sqrt{2x}\right)} \\
&= -\frac{1}{8}
\end{aligned}$$

□

b). *Observe que*

$$\begin{aligned}
B &= \lim_{x\to 0} \frac{\cos(\pi+x)+1}{x} \\
&= \lim_{x\to 0} \frac{\cos\pi\cos x - \operatorname{sen}\pi \operatorname{sen} x + 1}{x} \\
&= \lim_{x\to 0} \frac{-\cos x+1}{x}\frac{\cos x+1}{\cos x+1} \\
&= \lim_{x\to 0} \frac{1-\cos^2 x}{x}\frac{1}{\cos x+1} \\
&= \lim_{x\to 0} \frac{\operatorname{sen}^2 x}{x}\frac{1}{\cos x+1} \\
&= \lim_{x\to 0} \frac{\operatorname{sen} x}{x}\frac{\operatorname{sen} x}{\cos x+1} \\
&= 0
\end{aligned}$$

■

Exercício 2 *Sabe-se que*

$$f(x) = \frac{(a+b)x+(a-b)\,|x|}{2x}$$

Assim,

a).

$$\begin{aligned}
\lim_{x\to 0^-} f(x) &= \lim_{x\to 0^-} \frac{(a+b)x-(a-b)x}{2x} \\
&= \lim_{x\to 0^-} \frac{ax+bx-ax+bx}{2x} \\
&= \lim_{x\to 0^-} \frac{2bx}{2x} \\
&= b
\end{aligned}$$

□

b).

$$\begin{aligned}
\lim_{x\to 0^+} f(x) &= \lim_{x\to 0^+} \frac{(a+b)x+(a-b)x}{2x} \\
&= \lim_{x\to 0^+} \frac{ax+bx+ax-bx}{2x} \\
&= \lim_{x\to 0^+} \frac{2ax}{2x} \\
&= a
\end{aligned}$$

□

c). *Para que o*

$$\lim_{x\to 0} f(x)$$

exista é necessário que

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$$

Ou seja,

$$a = b$$

■

Exercício 3 *Observe que*

$$
\begin{aligned}
C &= \lim_{h\to 0}\frac{1-\cos 3h}{\cos^2 5h-1}\\
&= \lim_{h\to 0}\frac{1-\cos 3h}{\cos^2 5h-1}\frac{1+\cos 3h}{1+\cos 3h}\\
&= \lim_{h\to 0}\frac{1-\cos^2 3h}{-\left(1-\cos^2 5h\right)}\frac{1}{1+\cos 3h}\\
&= \lim_{h\to 0}\frac{\operatorname{sen}^2 3h}{-\operatorname{sen}^2 5h}\frac{1}{1+\cos 3h}\frac{\frac{(3h)^2}{(3h)^2}}{\frac{(5h)^2}{(5h)^2}}\\
&= \lim_{h\to 0}\frac{\frac{\operatorname{sen}^2 3h}{(3h)^2}}{-\frac{\operatorname{sen}^2 5h}{(5h)^2}}\frac{1}{1+\cos 3h}\frac{(3h)^2}{(5h)^2}\\
&= \lim_{h\to 0}\frac{\left(\frac{\operatorname{sen} 3h}{3h}\right)^2}{-\left(\frac{\operatorname{sen} 5h}{5h}\right)^2}\frac{1}{1+\cos 3h}\frac{9}{25}\\
&= -\frac{9}{50}
\end{aligned}
$$

■

Exercício 4 *Observe que*

$$
\begin{aligned}
D &= \lim_{x\to 8}\frac{\sqrt{7+\sqrt[3]{x}}-3}{x-8}\\
&= \lim_{x\to 8}\frac{\sqrt{7+\sqrt[3]{x}}-3}{x-8}\frac{\sqrt{7+\sqrt[3]{x}}+3}{\sqrt{7+\sqrt[3]{x}}+3}\\
&= \lim_{x\to 8}\frac{7+\sqrt[3]{x}-9}{x-8}\frac{1}{\sqrt{7+\sqrt[3]{x}}+3}\\
&= \lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}\frac{1}{\sqrt{7+\sqrt[3]{x}}+3}\\
&= \lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}\lim_{x\to 8}\frac{1}{\sqrt{7+\sqrt[3]{x}}+3}\\
&= M\cdot N
\end{aligned}
$$

Sendo

$$M = \lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}$$

e

$$
\begin{aligned}
N &= \lim_{x\to 8}\frac{1}{\sqrt{7+\sqrt[3]{x}}+3}\\
&= \frac{1}{6}
\end{aligned}
$$

Para resolver o limite M, considere

$$u = \sqrt[3]{x}$$

e perceba que

$$x = u^3$$

Além disso

$$x\to 8 \Rightarrow u\to 2$$

Logo

$$
\begin{aligned}
M &= \lim_{x\to 8}\frac{\sqrt[3]{x}-2}{x-8}\\
&= \lim_{u\to 2}\frac{u-2}{u^3-8}\\
&= \lim_{u\to 2}\frac{u-2}{(u-2)\left(u^2+2u+4\right)}\\
&= \lim_{u\to 2}\frac{1}{u^2+2u+4}\\
&= \frac{1}{12}
\end{aligned}
$$

E disto segue-se que

$$
\begin{aligned}
D &= M\cdot N\\
&= \frac{1}{12}\frac{1}{6}\\
&= \frac{1}{72}
\end{aligned}
$$

■

Exercício 5 *Deseja-se calcular o seguinte limite*

$$E = \lim_{x\to 0}\frac{\sqrt[3]{1+3x}-1}{x}$$

Para isto, considere

$$u = \sqrt[3]{1+3x}$$

e observe que

$$x = \frac{u^3-1}{3}$$

Além disto,

$$x\to 0 \Rightarrow u\to 1$$

Assim,

$$\begin{aligned}
E &= \lim_{x\to 0} \frac{\sqrt[3]{1+3x}-1}{x} \\
&= \lim_{u\to 1} \frac{u-1}{\frac{u^3-1}{3}} \\
&= \lim_{u\to 1} \frac{3\,(u-1)}{u^3-1} \\
&= \lim_{u\to 1} \frac{3\,(u-1)}{(u-1)\,(u^2+u+1)} \\
&= \lim_{u\to 1} \frac{3}{u^2+u+1} \\
&= 1
\end{aligned}$$

$\blacksquare$