Universidade Federal do Vale do São Francisco
Colegiado de Engenharia Civil
Cálculo Diferencial e Integral I

Prof^{o}. Edson

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Gabarito 1ª Prova **2015**
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Exercício 1

a). *Considere*

$$A = \lim_{x\to+\infty}\left(\sqrt{x^6+5} - x^3\right)$$

Observe que

$$A = \lim_{x\to+\infty}\left(\sqrt{x^6+5} - x^3\right)\frac{\sqrt{x^6+5}+x^3}{\sqrt{x^6+5}+x^3}$$

$$= \lim_{x\to+\infty}\frac{x^6+5-x^6}{\sqrt{x^6+5}+x^3}$$

$$= \lim_{x\to+\infty}\frac{5}{\sqrt{x^6+5}+x^3}$$

$$= 0$$

$\square$

b). *Observe que, quando $x\to+\infty$, tem-se que*

$$\frac{2}{x}\to 0$$

$$x+\frac{2}{x}\to+\infty$$

$$3x\to+\infty$$

Logo

$$\lim_{x\to+\infty}\left(x+\frac{2}{x}\right)^{3x} = +\infty$$

$\blacksquare$

Exercício 2

a). *Desejamos calcular o seguinte limite*

$$\lim_{x\to3}\frac{99^x-99^3}{x-3}$$

Para isto, considere

$$u = x-3$$

e observe que,

$$x\to3 \Rightarrow u\to0$$

e

$$x = u+3$$

Logo

$$\lim_{x\to3}\frac{99^x-99^3}{x-3} = \lim_{u\to0}\frac{99^{(u+3)}-99^3}{u}$$

$$= \lim_{u\to0}\frac{99^u99^3-99^3}{u}$$

$$= \lim_{u\to0}\frac{99^3\left(99^u-1\right)}{u}$$

Considere agora,

$$w = 99^u - 1$$

e observe que,

$$u\to0\Rightarrow w\to0$$

e

$$u = \log_{99}(w+1)$$

Assim,

$$\lim_{x\to3}\frac{99^x-99^3}{x-3} = \lim_{u\to0}\frac{99^3\left(99^u-1\right)}{u}$$

$$= 99^3\lim_{w\to0}\frac{w}{\log_{99}(w+1)}$$

$$= 99^3\lim_{w\to0}\frac{1}{\frac{1}{w}\log_{99}(w+1)}$$

$$= 99^3\lim_{w\to0}\frac{1}{\log_{99}(w+1)^{\frac{1}{w}}}$$

$$= 99^3\frac{1}{\log_{99}e}$$

$$= 99^3\ln 99$$

$\square$

b). *Desejamos agora, calcular o limite*

$$\lim_{x\to\frac{\pi}{4}} \frac{\operatorname{tg} x - 1}{x - \frac{\pi}{4}}$$

Tome

$$w = x - \frac{\pi}{4}$$

e observe que,

$$x \to \frac{\pi}{4} \Rightarrow w \to 0$$

e

$$x = w + \frac{\pi}{4}$$

Ou seja

$$\lim_{x\to\frac{\pi}{4}} \frac{\operatorname{tg} x - 1}{x - \frac{\pi}{4}} = \lim_{w\to 0} \frac{\operatorname{tg}(w + \frac{\pi}{4}) - 1}{w}$$

Temos da trigonometria, que

$$\operatorname{tg}(w + \frac{\pi}{4}) = \frac{\operatorname{sen}(w + \frac{\pi}{4})}{\cos(w + \frac{\pi}{4})}$$

$$= \frac{\operatorname{sen} w \, \cos\frac{\pi}{4} + \operatorname{sen}\frac{\pi}{4} \cos w}{\cos w \cos\frac{\pi}{4} - \operatorname{sen} w \operatorname{sen}\frac{\pi}{4}}$$

$$= \frac{\frac{\sqrt{2}}{2}\operatorname{sen} w + \frac{\sqrt{2}}{2}\cos w}{\frac{\sqrt{2}}{2}\cos w - \frac{\sqrt{2}}{2}\operatorname{sen} w}$$

$$= \frac{\frac{\sqrt{2}}{2}(\operatorname{sen} w + \cos w)}{\frac{\sqrt{2}}{2}(\cos w - \operatorname{sen} w)}$$

$$= \frac{\operatorname{sen} w + \cos w}{\cos w - \operatorname{sen} w}$$

Portanto,

$$\lim_{x\to\frac{\pi}{4}} \frac{\operatorname{tg} x - 1}{x - \frac{\pi}{4}} = \lim_{w\to 0} \frac{\operatorname{tg}(w + \frac{\pi}{4}) - 1}{w}$$

$$= \lim_{w\to 0} \frac{\frac{\operatorname{sen} w + \cos w}{\cos w - \operatorname{sen} w} - 1}{w}$$

$$= \lim_{w\to 0} \frac{\operatorname{sen} w + \cos w - \cos w + \operatorname{sen} w}{w(\cos w - \operatorname{sen} w)}$$

$$= \lim_{w\to 0} \frac{2\operatorname{sen} w}{w(\cos w - \operatorname{sen} w)}$$

$$= \lim_{w\to 0} 2\frac{\operatorname{sen} w}{w}\frac{1}{\cos w - \operatorname{sen} w}$$

$$= 2$$

■

Exercício 3 *Considere a função*

$$f(x) = \begin{cases} \dfrac{\operatorname{tg} kx}{x}, \text{ se } x < 0 \\ 3x + 2k^2, \text{ se } x \geq 0 \end{cases}$$

Para que a função f seja contínua em $x = 0$ devemos ter que

i). $f(0)$ *existe!*

Para isto, observe que

$$f(0) = 2k^2$$

ou seja, $f(0)$ depende da constante k, mas existe.

ii). $\lim_{x\to 0} f(x)$ *existe!*

Para isto, é necessário que

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x)$$

ou seja

$$\lim_{x\to 0^+}\left(3x+2k^2\right)=\lim_{x\to 0^-}\frac{\operatorname{tg} kx}{x}\Leftrightarrow$$

$$2k^2=\lim_{x\to 0^-}\frac{\operatorname{sen} kx}{x\cos kx}\Leftrightarrow$$

$$2k^2=\lim_{x\to 0^-}\frac{\operatorname{sen} kx}{kx}\frac{k}{\cos kx}$$

$$2k^2=k$$

$$2k^2-k=0$$

$$k=0\; ou\; \frac{1}{2}$$

iii). *Por fim,*

$$\lim_{x\to 0}f(x)=f(0)$$

ou seja, devemos ter mais uma vez que

$$2k^2=k\Leftrightarrow k=0\; ou\; \frac{1}{2}$$

Assim, como a constante k deve ser diferente de zero, temos que $k=\frac{1}{2}$. ■

Exercício 4

a).

$$\lim_{x\to 0}\frac{\sqrt{x^2+4}-2}{x^2}=\lim_{x\to 0}\frac{\sqrt{x^2+4}-2}{x^2}\frac{\sqrt{x^2+4}+2}{\sqrt{x^2+4}+2}$$

$$=\lim_{x\to 0}\frac{x^2+4-4}{x^2\left(\sqrt{x^2+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{x^2}{x^2\left(\sqrt{x^2+4}+2\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{x^2+4}+2}$$

$$=\frac{1}{4}$$

□

b). *Considere*

$$A=\lim_{x\to 0}\frac{2-\cos 3x-\cos 4x}{x}$$

Observe que

$$A=\lim_{x\to 0}\frac{1+1-\cos 3x-\cos 4x}{x}$$

$$=\lim_{x\to 0}\left(\frac{1-\cos 3x}{x}+\frac{1-\cos 4x}{x}\right)$$

$$=\lim_{x\to 0}\left(\frac{1-\cos 3x}{x}\frac{3}{3}+\frac{1-\cos 4x}{x}\frac{4}{4}\right)$$

$$=\lim_{x\to 0}\left(3\frac{1-\cos 3x}{3x}+4\frac{1-\cos 4x}{4x}\right)$$

$$=3\cdot 0+4\cdot 0$$

$$=0$$

■

Exercício 5

a). *Queremos calcular o limite*

$$\lim_{x\to +\infty}\left(1+\frac{3}{x}\right)^{-x}$$

Para isto, tome

$$w=\frac{x}{3}$$

e observe que

$$x\to +\infty\Rightarrow w\to +\infty$$

e

$$x=3w$$

Assim,

$$\lim_{x\to +\infty}\left(1+\frac{3}{x}\right)^{-x}=\lim_{w\to +\infty}\left(1+\frac{1}{w}\right)^{-3w}$$

$$=\lim_{w\to +\infty}\frac{1}{\left[\left(1+\frac{1}{w}\right)^{w}\right]^3}$$

$$=\frac{1}{e^3}$$

□

b). *Desejamos calcular*

$$B = \lim_{x \to -1} \frac{\operatorname{sen}\left(x^2 + 3x + 2\right)}{x^3 + 1}$$

Para isto, perceba que

$$x^2 + 3x + 2 = (x+1)(x+2)$$

e

$$x^3 + 1 = (x+1)(x^2 - x + 1)$$

Logo,

$$\begin{aligned} B &= \lim_{x \to -1} \frac{\operatorname{sen}\left[(x+1)(x+2)\right]}{(x+1)(x^2 - x + 1)} \\ &= \lim_{x \to -1} \frac{(x+2)\operatorname{sen}\left[(x+1)(x+2)\right]}{(x+2)(x+1)(x^2 - x + 1)} \\ &= \lim_{x \to -1} \frac{\operatorname{sen}\left[(x+1)(x+2)\right]}{(x+1)(x+2)} \frac{(x+2)}{(x^2 - x + 1)} \\ &= 1 \cdot \frac{1}{3} \\ &= \frac{1}{3} \end{aligned}$$

$\blacksquare$