Universidade Federal do Vale do São Francisco
Engenharia Civil
Cálculo Diferencial e Integral I

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Gabarito 2ª Prova **2005**
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Exercício 1

a).

$$\begin{aligned}
f(x) &= \ln\left(\frac{\cos\sqrt{x}}{1+\text{sen}\sqrt{x}}\right) \Rightarrow \\
f'(x) &= \frac{1}{\frac{\cos\sqrt{x}}{1+\text{sen}\ \sqrt{x}}} \cdot \left(\frac{\cos\sqrt{x}}{1+\text{sen}\ \sqrt{x}}\right)' \\
&= \frac{1+\text{sen}\sqrt{x}}{\cos\sqrt{x}} \cdot \frac{-\frac{\text{sen}\sqrt{x}}{2\sqrt{x}}\left(1+\text{sen}\ \sqrt{x}\right)-\frac{\cos^2\sqrt{x}}{2\sqrt{x}}}{\left(1+\text{sen}\ \sqrt{x}\right)^2} \\
&= \frac{1}{\cos\sqrt{x}} \cdot \frac{-\frac{1}{2\sqrt{x}}\left[\text{sen}\ \sqrt{x}+\text{sen}^2\sqrt{x}+\cos^2\sqrt{x}\right]}{1+\text{sen}\ \sqrt{x}} \\
&= \frac{1}{\cos\sqrt{x}} \cdot \frac{-\frac{1}{2\sqrt{x}}\left[\text{sen}\ \sqrt{x}+1\right]}{1+\text{sen}\ \sqrt{x}} \\
&= \frac{1}{\cos\sqrt{x}} \cdot \left(-\frac{1}{2\sqrt{x}}\right) \\
&= \frac{-1}{2\sqrt{x}\cos\sqrt{x}}
\end{aligned}$$

□

b).

$$\begin{aligned}
g(x) &= \ln\left(\text{sen}\ 5x\right) \Rightarrow \\
g'(x) &= \frac{1}{\text{sen}\ 5x} \cdot \left(\text{sen}\ 5x\right)' \\
&= \frac{1}{\text{sen}\ 5x} 5\cos 5x \\
&= 5\frac{\cos 5x}{\text{sen}\ 5x} \\
&= 5\cot\text{g}\ 5x
\end{aligned}$$

■

Exercício 2

a). Tome $x = y = 0$ e observe que

$$\begin{aligned}
f(0) &= f(0) + f(0) \Rightarrow \\
f(0) &= 2f(0) \Rightarrow \\
f(0) - 2f(0) &= 0 \Rightarrow \\
-f(0) &= 0 \Rightarrow \\
f(0) &= 0
\end{aligned}$$

□

b).

$$f'(0) = \lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{f(h)}{h} = 1$$

□

c).

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{f(x)+f(h)+x^2h+xh^2-f(x)}{h} \\
&= \lim_{h\to 0}\frac{f(h)+x^2h+xh^2}{h} \\
&= \lim_{h\to 0}\left[\frac{f(h)}{h}+x^2+xh\right] \\
&= 1+x^2
\end{aligned}$$

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Exercício 3 *Queremos calcular a derivada da função*

$$g(x) = \text{arcsec}\, x$$

Usando a regra da inversa, temos que

$$g'(x) = \frac{1}{f'(g(x))}$$

onde $f(x) = \sec x$ é a inversa de g; Calculando f' temos:

$$\begin{aligned}
f(x) &= \sec x = \frac{1}{\cos x} \Rightarrow \\
f'(x) &= \frac{\text{sen}\ x}{\cos^2 x} = \frac{\text{sen}\ x}{\cos x}\frac{1}{\cos x} \\
&= \sec x \text{tg}\ x
\end{aligned}$$

Logo

$$\begin{aligned}
g'(x) &= \frac{1}{\sec\left(\text{arcsec}\, x\right)\text{tg}\left(\text{arcsec}\, x\right)} \\
&= \frac{1}{x\text{tg}\left(\text{arcsec}\, x\right)}
\end{aligned}$$

Usando que

$$\text{tg}\ x = \sqrt{\sec^2 x - 1}, \forall x \in \mathbb{R}$$

segue-se que:

$$g'(x) = \frac{1}{x\sqrt{\sec^2\left(\text{arcsec}\, x\right)-1}} = \frac{1}{x\sqrt{x^2-1}}$$

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Exercício 4

a). Considere as funções

$$p(x) = x \text{ e } q(x) = x^3$$

Como p e q são deriváveis em $\mathbb{R}$ e f é derivável em $I \subset \mathbb{R}$, podemos afirmar que

i). $q(f(x)) = [f(x)]^3$ *é derivável em I;*

ii). $p(x) + q(f(x) = x + [f(x)]^3$ *é derivável em I;*

Logo

$$f'(x) = x + [f(x)]^3$$

é derivável em $I \subset \mathbb{R}$. □

b).

$$\begin{aligned} f''(x) &= [f'(x)]' \\ &= \left[x + [f(x)]^3\right]' \\ &= 1 + 3f(x)^2 f'(x) \Rightarrow \\ \\ f''(1) &= 1 + 3f(1)^2 f'(1) \\ &= 1 + 3\left[1 + f(1)^2\right] \\ &= 1 + 3 \cdot 2 \\ &= 7. \end{aligned}$$

□

c). A reta que passa por $(1, f(1)) = (1,1)$ e tem coeficiente angular $m = f'(1) = 2$ é dada por

$$y - 1 = 2(x-1) \Rightarrow y = 2x - 1$$

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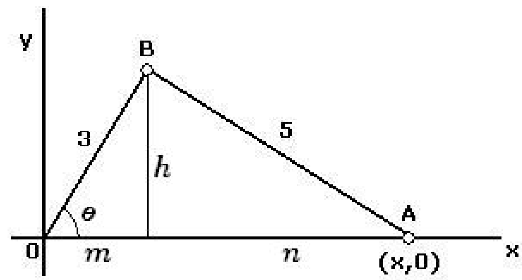
Exercício 5 *Considere a figura:*



Temos que:

$$\text{sen } \theta = \frac{h}{3} \quad \Rightarrow \quad h = 3 \text{ sen } \theta$$

$$\cos\theta = \frac{m}{3} \quad \Rightarrow \quad m = 3\cos\theta$$

$$h^2 + n^2 = 25 \quad \Rightarrow \quad n = \sqrt{25 - 9 \text{ sen}^2 \theta}$$

$$x = m + n \quad \Rightarrow \quad x(\theta) = 3\cos\theta + \sqrt{25 - 9 \text{ sen}^2 \theta}$$

Logo

$$\begin{aligned} \frac{dx}{dt} &= \frac{dx}{d\theta}\frac{d\theta}{dt} \\ &= \left(-3 \text{ sen } \theta - \frac{\theta \text{sen } \theta \cos\theta}{\sqrt{25 - 9 \text{ sen}^2 \theta}}\right)\frac{d\theta}{dt} \end{aligned}$$

e quando $\theta = \frac{\pi}{2}$ *e* $\frac{d\theta}{dt} = \frac{1}{2}$, *teremos*

$$\frac{dx}{dt} = \frac{-3}{2} cm/s$$

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